

Imagine you can buy and sell these contracts on Reddit, e.g.

(It's external to the Portfolio app in our story.)

Every derivative security can be modeled

by a r.v. $V_1 = V_1(\xi_1)$

the dollar amount the seller must give the buyer (depends on ξ_1)

Conversely, every r.v. describes some derivative security.

Defn N1.1.3 A derivative security

is a r.v. $V_1 = V_1(\xi_1)$

Expl

European call option w/strike price K

Gives the buyer of the contract the option (but not the obligation) to buy 1 share (from the seller) at time 1 for price K .

$$V_1 = \begin{cases} S_1 - K & \text{if } S_1 > K \\ 0 & \text{if } S_1 \leq K \end{cases}$$

↑ buyer won't exercise the option in this case

More compactly: $V_1 = (S_1 - K)^+$

Expl

European put option w/strike price K

Gives the buyer of the contract the option (but not the obligation) to sell 1 share (to the seller of the contract) at time 1 for price K .

$$V_1 = \begin{cases} K - S_1 & \text{if } S_1 < K \\ 0 & \text{if } S_1 \geq K \end{cases}$$

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↑ buyer won't exercise the option in this case

More compactly: $V_1 = (K - S)^+$

Expl

Forward contract w/strike price K

The buyer of the contract commits to buying 1 share from the seller at time 1 for price K (regardless of the market price at that time). In this case,

$$V_1 = S_1 - K$$

Expl

$N=1$, $S_0=4$, $u=2$, $d=\frac{1}{2}$, $r=\frac{1}{4}$, $p=\text{any}$

I sell a European call option with strike price $K=5$ to stockburn42 on Reddit.

He gives me \$1.20 for it.

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I start to worry that if the stock goes up, I'll lose \$3, for a net loss of \$1.80.

I ask for my contract back but he refuses.

What can I do?

ANSWER: Open Portfolio app and execute the strategy $(X_0, \Delta_0) = (1.2, \frac{1}{2})$. (What does this mean in practical terms?)

$$X_1 = (1 + \frac{1}{4}) (\cancel{X_0} - \cancel{\Delta_0} S_0) + \cancel{\Delta_0} S_1$$
$$= \frac{5}{4} (-\frac{4}{5}) + \frac{1}{2} \cancel{X_1} S_0$$

$$= -1 + 2\gamma_1 = \begin{cases} -1 + 2\cancel{\mu}^2 & \text{if } \xi_1 = 1 \\ -1 + 2\cancel{\mu}^{\frac{1}{2}} & \text{if } \xi_1 = -1 \end{cases}$$

$$= \begin{cases} 3 & \text{if } \xi_1 = 1 \\ 0 & \text{if } \xi_1 = -1 \end{cases}$$

At time 1, I will owe

$$V_1 = (S_1 - \overset{5}{K})^+ = \begin{cases} 3 & \text{if } \xi_1 = 1 \\ 0 & \text{if } \xi_1 = -1 \end{cases}$$

So $X_1 = V_1$.

Starting w/ \$1.20 in the Portfolio app, I can trade strategically so that I can use final value in the Portfolio to cover the contract I sold.

In general, if (X_0, Δ_0) produces an X_1 such that $X_1 = V_1$, then (X_0, Δ_0) replicates V_1 .

In the previous expl, $(1.2, \frac{1}{2})$ replicates $(S_1 - 5)^+$.

Expl Reverse the story of the previous expl.

stockburn42 gets nervous.

He asks for his money back. I refuse.

What can he do?

ANSWER: Use the strategy $(-1.2, -\frac{1}{2})$.

He immediately get \$1.20 in his bank from the Portfolio app (that's his money back). Then...

$$X_1 = (1 + \frac{1}{4}) (X_0 - \frac{-1.2}{2} S_0) + \frac{-1}{2} S_1$$

$$= \frac{5}{4} (0.8) - \frac{1}{2} Y_1 S_0$$

$$= 1 - 2Y_1 = \begin{cases} 1 - 2 \times \frac{2}{2} & \text{if } \xi_1 = 1 \\ 1 - 2 \times \frac{1}{2} & \text{if } \xi_1 = -1 \end{cases}$$

$$= \begin{cases} -3 & \text{if } \xi_1 = 1 \\ 0 & \text{if } \xi_1 = -1 \end{cases}$$

So for him, $X_1 = -V_1$. But he will get V_1 .

So in the end, he is even.

Expl What if stockbunt2 is willing to pay \$2 for the option? What can I do?

ANSWER: Sell 1 option and use $(X_0, \Delta_0) = (2, \frac{1}{2})$.

IDEA: $(X_0, \Delta_0) = (2, \frac{1}{2}) = (1.2, \frac{1}{2}) + (0.8, 0)$ ←

covers my contractual obligations ↗

gets me free money:
 $(1+r)(0.8) = \frac{5}{4} (\frac{4}{5}) = \$1$

VERIFY: In the end, I will have $X_1 - V_1$

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$$X_1 = (1+r)(X_0 - \Delta_0 S_0) + \Delta_0 S_1$$
$$= \frac{5}{4} \left(2 - \frac{1}{2}(4) \right) + \frac{1}{2} S_1 = \begin{cases} 4 & \text{if } \xi_1 = 1 \\ 1 & \text{if } \xi_1 = -1 \end{cases}$$

$$V_1 = \begin{cases} 3 & \text{if } \xi_1 = 1 \\ 0 & \text{if } \xi_1 = -1 \end{cases}$$

So $X_1 - V_1 = 1$ (no matter what ξ_1 is)
 $\uparrow$ free money

Conversely, what if I am willing to sell the option for \$1? What can stock buyer do?

ANSWER: Buy 1 option and use $(X_0, \Delta_0) = (-1, -\frac{1}{2})$

IDEA: $(X_0, \Delta_0) = (-1, -\frac{1}{2}) = (-1.2, -\frac{1}{2}) + (0.2, 0)$
gets him his money back $\rightarrow$ gets him free money:
 $(1+r)(0.2) = \frac{5}{4} \left(\frac{1}{5} \right) = \$0.25$

VERIFY: In the end, he will have $X_1 + V_1$

$$X_1 = (1+r)(X_0 - \Delta_0 S_0) + \Delta_0 S_1$$
$$= \frac{5}{4} \left(-1 - \left(-\frac{1}{2} \right) (4) \right) - \frac{1}{2} S_1 = \frac{5}{4} - \frac{1}{2} S_1 = \begin{cases} -\frac{11}{4} & \text{if } \xi_1 = 1 \\ \frac{1}{4} & \text{if } \xi_1 = -1 \end{cases}$$

$$V_1 = \begin{cases} 3 & \text{if } \xi_1 = 1 \\ 0 & \text{if } \xi_1 = -1 \end{cases}$$

$$X_1 + V_1 = \frac{1}{4} \quad (\text{no matter what } \xi_1 \text{ is})$$

↑ free money

So \$1.20 is the "right" price. With any other price, someone is getting free money.

(Notice that none of this depends on p .)

Thm N1.1.4 Given a derivative security V_1 , there exists a unique strategy (X_0, Δ_0) that replicates it.

(We'll prove this later.)

Defn N1.1.5 Let V_1 be a derivative security. Let (X_0, Δ_0) be its unique replicating strategy. The (no-arbitrage) price of the security at time 0 is X_0 .

NOTE: We often use V_0 to denote the price 37 of V_1 at time 0. With that notation, the definition says that if (X_0, Δ_0) replicates V_1 , then $V_0 = X_0$.

To see that every V_1 can be replicated, can simply check that this strategy does it:

$$X_0 = \frac{1}{1+r} \left(\frac{1+r-d}{u-d} V_1(1) + \frac{u-(1+r)}{u-d} V_1(-1) \right)$$

$$\Delta_0 = \frac{V_1(1) - V_1(-1)}{S_1(1) - S_1(-1)} = \frac{V_1(1) - V_1(-1)}{(u-d)S_0}.$$

Can simplify X_0 :

$$\tilde{p} := \frac{1+r-d}{u-d}, \quad \tilde{q} := \frac{u-(1+r)}{u-d} = 1 - \tilde{p}$$

$\tilde{P} = P^{\tilde{P}}$, use E for E^P and $\tilde{E}$ for $E^{\tilde{P}}$

$$X_0 = \frac{1}{1+r} \tilde{E}[V_1]$$

$\tilde{p}, \tilde{q}$: risk-neutral probabilities
 $\tilde{P}$: risk-neutral probability measure
 → artificial, defined for mathematical convenience

Noteworthy fact about $\tilde{P}$:

$$\tilde{E}[S_1] = \tilde{p} S_{1,(1)} + \tilde{q} S_{1,(-1)}$$

$$= \tilde{p} u S_0 + \tilde{q} d S_0 = (\tilde{p} u + \tilde{q} d) S_0$$

a lot of algebra → $\stackrel{\text{a lot of algebra}}{=} (1+r) S_0$

Under risk-neutral probabilities,

growth of mm = average growth of stock

$\tilde{p}, \tilde{q}$ characterized by
 $\tilde{p} u + \tilde{q} d = 1+r$

Now check that

$$X_0 = \frac{1}{1+r} \tilde{E}[V_1] = \frac{1}{1+r} (\tilde{p} V_{1,(1)} + \tilde{q} V_{1,(-1)})$$

$$\Delta_0 = \frac{V_{1,(1)} - V_{1,(-1)}}{(u-d) S_0}$$

replicates V_1 . Need to check that $X_1 = V_1$.

So need to check $X_{1,(1)} = V_{1,(1)}$ and $X_{1,(-1)} = V_{1,(-1)}$.

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$$X_1 = (1+r)(X_0 - \Delta_0 S_0) + \Delta_0 S_1$$

$$= (1+r)X_0 + \Delta_0(S_1 - (1+r)S_0)$$

$$= \tilde{p} V_1(1) + \tilde{q} V_1(-1) + \Delta_0(S_1 - (1+r)S_0)$$

$$S_1(1) = uS_0 \Rightarrow$$

$$X_1(1) = \tilde{p} V_1(1) + \tilde{q} V_1(-1) + \Delta_0 \overset{\tilde{q}(u-d)}{\cancel{(u-(1+r))}} S_0$$

$$\Delta_0(u-d)S_0 = V_1(1) - V_1(-1) \Rightarrow$$

$$\begin{aligned} X_1(1) &= \tilde{p} V_1(1) + \cancel{\tilde{q} V_1(-1)} + \tilde{q} V_1(1) - \cancel{\tilde{q} V_1(-1)} \\ &= (\tilde{p} + \tilde{q}) V_1(1) = V_1(1) \checkmark \\ &\quad \underline{1} \end{aligned}$$

Checking $X_1(-1) = V_1(-1)$ is similar.

So every V_i can be replicated. (Still need to prove replicating strategy is unique. Will do that in §2.4.)

§1.2 Multiperiod binomial model

Recall the Portfolio app story. We now consider an N -day plan

Stock price is modeled w/binomial model: (40)

$$N = \text{any}, S_0 > 0, 0 < d < u, p \in (0, 1)$$

$$\Omega = \Omega_N = \{-1, 1\}^N$$

$$= \{\omega = (\omega_1, \dots, \omega_N) : \text{each } \omega_j = -1 \text{ or } 1\}$$

$\xi_1, \dots, \xi_N$: results of coin flips (independent)

$$\xi_n(\omega) = \omega_n$$

α : # of heads among the N flips

$$\alpha(\omega) = |\{n : \omega_n = 1\}|$$

$P = P^p$ (probability measure)

$$P(\omega) = p^{\alpha(\omega)} q^{N-\alpha(\omega)}$$

modeling the money market

r : daily interest rate

$\$a$ on Day n becomes $\$(1+r)^m a$ on Day $(n+m)$
(true whether $a > 0$, $a = 0$, or $a < 0$)

assume $r > -1$ and $\underbrace{d < 1+r < u}$

no arbitrage assumption

modeling the investor's choices:

4(a)

X_0 : amount of initial transfer
↳ initial wealth

If $\Delta_0 = 3, \Delta_1 = 1$, then
buy 3 on Day 0 and
sell 2 on Day 1.

Δ_n : # of shares held on Day n
($n = 0, 1, 2, \dots, N-1$)

Δ_0 is a constant (not random)

for $n > 0$, $\Delta_n = \Delta_n(\xi_1, \dots, \xi_n)$
(can use 1st n coin flips to
decide on Δ_n)

$\Delta = (\Delta_0, \Delta_1, \Delta_2, \dots, \Delta_{N-1})$

(X_0, Δ) called a trading strategy

In this class, a stochastic process is a finite
sequence of random variables, $Z = (Z_0, \dots, Z_N)$.

(In general, a stochastic process is any indexed
collection of r.v.s)

So Δ is a stochastic process (w/o a Δ_N).

A stochastic process Z is adapted if Z_0 is a constant (i.e. not random) and Z_n is a function of $\xi_1, \dots, \xi_n$ for all $n > 0$.

So Δ is adapted.

X_n = value of everything in Portfolio app when you wake up on Day n .
called wealth at time n

$$X = (X_0, X_1, X_2, \dots, X_N)$$

↑
wealth process

$$X_{n+1} = (1+r) \underbrace{(X_n - \Delta_n S_n)}_{\text{amount in mm at end of Day } n} + \underbrace{\Delta_n S_{n+1}}_{\text{value of shares at start of Day } n+1}$$

OR

$$X_{n+1} = \underbrace{X_n}_{\text{previous wealth}} + \underbrace{r(X_n - \Delta_n S_n)}_{\text{interest from mm}} + \underbrace{\Delta_n (S_{n+1} - S_n)}_{\text{profit/loss from trading}}$$

Defn N1.2.1 An arbitrage is a trading strategy 42

(X_0, Δ) such that

(i) $X_0 = 0$,

(ii) $P(X_N < 0) = 0$, and

(iii) $P(X_N > 0) > 0$.

Thm N1.2.2 There is no arbitrage in the N -period binomial model.

(We will prove this in §2.4.)

Derivative securities are contracts like before, but are exercised at time N instead of time 1.

Defn N1.2.3: A derivative security

is a r.v. $V_N = V_N(\xi_1, \dots, \xi_N)$

↑ the amount the contract pays the buyer at time N .

If the trading strategy (X_0, Δ) produces a final wealth X_N satisfying $X_N = V_N$, then (X_0, Δ) replicates V_N .

Thm N1.2.4 Given a derivative security V_N , there exists a unique strategy (X_0, Δ) that replicates it.

(We'll prove this later. Note that Thm N1.1.4 is a special case of this.)

To see that every V_N can be replicated, we construct a replicating strategy.

Let $\tilde{p}, \tilde{q}$ be the risk-neutral probabilities. $\tilde{P} = P^{\tilde{p}}$, use E for E^P and $\tilde{E}$ for $E^{\tilde{P}}$

Let V_N be an arbitrary derivative.

For $n < N$, define

$$V_n = \frac{1}{1+r} \tilde{\mathbb{E}}_n [V_{n+1}]$$

Note that $\tilde{\mathbb{E}}_0 = \tilde{\mathbb{E}}$, so that V_0 will be a non-random constant.

Now define $X_0 = V_0$,

$$\Delta_n(\xi_1, \dots, \xi_n) = \frac{V_{n+1}(\xi_1, \dots, \xi_n, 1) - V_{n+1}(\xi_1, \dots, \xi_n, -1)}{S_{n+1}(\xi_1, \dots, \xi_n, 1) - S_{n+1}(\xi_1, \dots, \xi_n, -1)}$$

and $\Delta = (\Delta_0, \Delta_1, \Delta_2, \dots, \Delta_{N-1})$.

Thm 1.2.2

The strategy (X_0, Δ) above replicates V_N .

In fact, when using this strategy, we have

$X_n = V_n$ for all $0 \leq n \leq N$.

Proof: Calculate

$$V_n = \frac{1}{1+r} \tilde{\mathbb{E}}_n [V_{n+1}] = \frac{1}{1+r} g(\xi_1, \dots, \xi_n),$$

where

$$\begin{aligned} g(x_1, \dots, x_n) &= \mathbb{E}[V_{n+1}(x_1, \dots, x_n, \xi_{n+1})] \\ &= \tilde{p} V_{n+1}(x_1, \dots, x_n, 1) + \tilde{q} V_{n+1}(x_1, \dots, x_n, -1). \end{aligned}$$

So

$$V_n = \frac{1}{1+r} \left(\tilde{p} V_{n+1}(\xi_1, \dots, \xi_n, 1) + \tilde{q} V_{n+1}(\xi_1, \dots, \xi_n, -1) \right)$$

↑ use this formula to compute V_n in problems and applications in this section

From here, you can read in the book the proof that $X_n = V_n$ for all n . $\square$

Thm 1.2.2 proves Thm N1.2.4 except for uniqueness. Uniqueness will be proven in § 2.4.

Defn 1.2.3 The (no-arbitrage) price of the derivative security V_N at time 0 is the quantity V_0 defined above.

The (no-arbitrage) price of the derivative security V_N at time n , given $\xi_1, \dots, \xi_n$ is the r.v. V_n defined above.

Why is V_n the price of V_N at time n ?

Suppose you're following (X'_0, Δ') . At time n , you have an opportunity to buy V_N . You happen to have $X_n = V_n$ at time n . If (X_0, Δ)

replicates V_N , can you switch to Δ at time n and still achieve $X_N = V_N$? YES

$$X_{n+1} = (1+r)(X_n - \Delta_n S_n) + \Delta_n S_{n+1}$$

shows that $(X_{n+1}, \dots, X_N)$ depends only on X_n and $(\Delta_n, \dots, \Delta_{N-1})$.

Since every derivative security can be replicated, we call the model complete. (466)

If $V_N = h(S_N)$ for some h , then V_N is a path-independent option. Otherwise V_N is a path-dependent option.

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Expl

$$N=2, S_0=4, u=2, d=\frac{1}{2}, r=\frac{1}{4}, p=\text{any}$$

$$V_2 = (S_2 - 5)^+ \quad (\text{Euro. call w/ strike price } K=5)$$

(This is path-independent.)

Find the replicating strategy.

$$V_1 = \frac{1}{1+r} (\tilde{p} V_2(\xi_{1,1}) + \tilde{q} V_2(\xi_{1,-1}))$$

$$\tilde{p} = \frac{1+r-d}{u-d} = \frac{\frac{5}{4} - \frac{1}{2}}{2 - \frac{1}{2}} = \frac{\frac{3}{4}}{\frac{3}{2}} = \frac{3}{4} \cdot \frac{2}{3} = \frac{1}{2}$$

$$\tilde{q} = 1 - \tilde{p} = \frac{1}{2}$$

$$\frac{1}{1+r} = \frac{1}{\frac{5}{4}} = \frac{4}{5}$$

$$\begin{aligned} V_1 &= \frac{4}{5} \left(\frac{1}{2} V_2(\xi_{1,1}) + \frac{1}{2} V_2(\xi_{1,-1}) \right) \\ &= \frac{2}{5} V_2(\xi_{1,1}) + \frac{2}{5} V_2(\xi_{1,-1}) \end{aligned}$$

$$V_1 = V_1(\xi_1)$$

$$V_1(1) = \frac{2}{5} V_2(1,1) + \frac{2}{5} V_2(1,-1)$$

$$V_2(1,1) = (S_2(1,1) - 5)^+ = (16 - 5)^+ = 11$$

$$V_2(1,-1) = (S_2(1,-1) - 5)^+ = (4 - 5)^+ = 0$$

$$V_1(1) = \frac{2}{5} \cdot 11 + \frac{2}{5} \cdot 0 = \frac{22}{5}$$

$$\begin{aligned} V_1(-1) &= \frac{2}{5} V_2(-1,1) + \frac{2}{5} V_2(-1,-1) \\ &= \frac{2}{5} (\cancel{S_2(-1,1)} - 5)^+ + \frac{2}{5} (\cancel{S_2(-1,-1)} - 5)^+ = 0 \end{aligned}$$

$$V_1 = \begin{cases} \frac{22}{5} & \text{if } \xi_1 = 1 \\ 0 & \text{if } \xi_1 = -1 \end{cases}$$

$$\begin{aligned} V_0 &= \frac{1}{1+r} (\tilde{p} V_1(1) + \tilde{q} V_1(-1)) \\ &= \frac{4}{5} \left(\frac{1}{2} \cdot \frac{22}{5} + \frac{1}{2} \cdot 0 \right) = \frac{44}{25} \end{aligned}$$

$$X_0 = V_0 = \frac{44}{25}$$

$$\Delta = ? \quad , \quad \Delta = (\Delta_0, \Delta_1)$$

$$\Delta_n(\xi_1, \dots, \xi_n) = \frac{V_{n+1}(\xi_1, \dots, \xi_n, 1) - V_{n+1}(\xi_1, \dots, \xi_n, -1)}{S_{n+1}(\xi_1, \dots, \xi_n, 1) - S_{n+1}(\xi_1, \dots, \xi_n, -1)}$$

$$\Delta_0 = \frac{V_1(1) - V_1(-1)}{S_1(1) - S_1(-1)} = \frac{\frac{22}{5} - 0}{8 - 2} = \frac{11}{5} \cdot \frac{1}{6} = \frac{11}{15}$$

$$\Delta_1 = \frac{V_2(\xi_1, 1) - V_2(\xi_1, -1)}{S_2(\xi_1, 1) - S_2(\xi_1, -1)}$$

$$\Delta_1(1) = \frac{\overset{11}{\cancel{V_2(1,1)}} - \overset{0}{\cancel{V_2(1,-1)}}}{\underset{16}{\cancel{S_2(1,1)}} - \underset{4}{\cancel{S_2(1,-1)}}} = \frac{11}{12}$$

$$\Delta_1(-1) = \frac{\overset{0}{\cancel{V_2(-1,1)}} - \overset{0}{\cancel{V_2(-1,-1)}}}{\underset{4}{\cancel{S_2(-1,1)}} - \underset{1}{\cancel{S_2(-1,-1)}}} = 0$$

$$\Delta_1 = \begin{cases} \frac{11}{12} & \text{if } \xi_1 = 1 \\ 0 & \text{if } \xi_1 = -1 \end{cases}$$

Replicating strategy: (X_0, Δ)

$$X_0 = \frac{44}{25} = 1.76$$

$$\Delta = (\Delta_0, \Delta_1)$$

$$\Delta_0 = \frac{11}{15}, \quad \Delta_1 = \begin{cases} \frac{11}{12} & \text{if } \xi_1 = 1 \\ 0 & \text{if } \xi_1 = -1 \end{cases}$$

Let's try and describe this in words.

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Make an initial transfer into the Portfolio app of \$1.76.

mm: 1.76	$S_0 = 4$	Day 0 morning
shares: 0		

Immediately buy $\frac{11}{15}$ shares for

$$4\left(\frac{11}{15}\right) = \frac{44}{15} = \$2.93$$

mm: -1.17	$S_0 = 4$	Day 0 night
shares: $\frac{11}{15}$		

Go to sleep. Wake up to one of 2 possibilities:

$\text{mm: } -1.47$ $\text{shares: } \frac{11}{15}$ $S_1 = 8$ $\xi_1 = 1$	$\text{mm: } -1.47$ $\text{shares: } \frac{11}{15}$ $S_1 = 2$ $\xi_1 = -1$	Day 1 morning
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$$\downarrow \Delta_1(1) = \frac{11}{12}$$

$$\left(\text{buy } \frac{11}{12} - \frac{11}{15} = \frac{11}{60} \text{ shares} \right)$$

$$\left(\text{for } 8\left(\frac{11}{60}\right) = \$1.47 \right)$$

$$\downarrow \Delta_1(-1) = 0$$

$$\left(\text{sell } \frac{11}{15} \text{ shares for} \right)$$

$$\left(2\left(\frac{11}{15}\right) = \$1.47 \right)$$

$\text{mm: } -2.93$ $\text{shares: } \frac{11}{12}$ $S_1 = 8$	$\text{mm: } 0$ $\text{shares: } 0$ $S_1 = 2$	Day 1 night
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Sleep. Wake up to...

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mm: -2.93 shares: $\frac{11}{12}$ $S_1 = 8$	mm: 0 shares: 0 $S_1 = 2$
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Day 1
night

$\xi_2 = 1$

$\xi_2 = -1$

mm: -3.67
shares: $\frac{11}{12}$
 $S_2 = 16$
Wealth: X_2
 $= \frac{11}{12}(16) - 3.67$
 $= \$11$

mm: -3.67
shares: $\frac{11}{12}$
 $S_2 = 4$
Wealth: X_2
 $= \frac{11}{12}(4) - 3.67$
 $= \$0$

mm: 0
shares: 0
 $S_1 = 4 \text{ or } 1$
Wealth: $X_2 = \$0$

The Euro. call is $V_2 = (S_2 - 5)^+$. So...

$$S_2 = 16$$

↓

$$V_2 = 11$$

$$S_2 = 4$$

↓

$$V_2 = 0$$

$$S_2 = 4 \text{ or } 1$$

↓

$$V_2 = 0$$

So $X_2 = V_2$ and we have replicated the Euro. call.

Expl 1.2.4

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$$N=3, S_0=4, u=2, d=\frac{1}{2}, r=\frac{1}{4}, p=\text{any}$$

$$M_3 = \max_{0 \leq n \leq 3} S_n, \quad V_3 = M_3 - S_3$$

(a) Find the price of this derivative at time 0.

(b) Consider an investor that is replicating

(or hedging) this derivative. How many

shares do they buy on Day 0? (Give a

negative number if they should sell instead.)

Describe their portfolio at the end of Day 0.

(c) Suppose the stock goes up on Day 1

(meaning $S_1 > S_0$).

How many shares should they buy on Day 1?

Describe their portfolio at the end of Day 1.

(d) Suppose the stock then goes down on Day 2.

How many shares should they buy on Day 2?

Describe their portfolio at the end of Day 2.