

$= 362880$ ← # of ways to "permute" the players
i.e. # of "permutations"

Defn $0! = 1$,
 $n! = n \cdot (n-1)!$ for $n \geq 1$
↑ factorial

$n! = \underbrace{n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1}_{\text{\# of permutations of } n \text{ objects}}$

Expl 1.3d

How many arrangements of the letters in PEPPER are possible?

Permutations require distinct objects

$P_1 E_1 P_2 P_3 E_2 R$ ← $6!$ permutations of these.
But that's too many

e.g. $\left. \begin{array}{l} P_1 P_2 E_1 P_3 E_2 R \\ P_3 P_1 E_2 P_2 E_1 R \end{array} \right\}$ both are just PPEPER

How many of the $6!$ correspond to PPEPER?

Answer: $3!2!$
 ↑ # of ways to order the P's # of ways to order the E's

Same is true for ERPEPP or any other arrangement.

So final answer is

$$\frac{6!}{3!2!} = \frac{\cancel{6} \cdot 5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot 1}{\cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{2} \cdot 1} = 5 \cdot 4 \cdot 3 = \boxed{60}$$

Expl 1.3f

4W, 3R,
2B

→ put all 9 in a line
How many arrangements?

Same reasoning.

$$\frac{9!}{4!3!2!} = \frac{9 \cdot \cancel{8} \cdot 7 \cdot \cancel{6} \cdot 5 \cdot \cancel{4} \cdot 3 \cdot 2}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{2} \cdot 2}$$

$$= 9 \cdot 7 \cdot 5 \cdot 4 = 63 \cdot 20 = \boxed{1260}$$

Expl

How many 4 digit #'s are there with distinct digits and no 0's?

$$9 \cdot 8 \cdot 7 \cdot 6 = \boxed{3024}$$

↑ choose 1st digit
 ↑ choose 2nd
 ↑ choose 3rd
 ↑ choose 4th

$$\frac{n!}{(n-r)!} = \frac{n(n-1)(n-2) \cdots (n-r+1)(n-r)(n-r-1) \cdots 3 \cdot 2 \cdot 1}{\cancel{(n-r)(n-r-1) \cdots 3 \cdot 2 \cdot 1}}$$

$$= \underbrace{n(n-1)(n-2) \cdots (n-r+1)}_{\text{\# of factors : } n - (n-r) = r}$$

of ways to choose r distinct objects from n possible objects, if order matters.

1.4 Combinations

Expl 1.4a

20 people → committee of 3
 How many committees?

$20 \cdot 19 \cdot 18$ ← too many.
 ↑ choose 1st member ↑ choose 2nd ↑ choose 3rd
 order chosen doesn't matter

e.g. ABC, ACB, BAC, BCA, CAB, CBA

are all the same committee

Need to divide by 6, which is $3!$
of ways to permute the choices

$$\text{Final answer: } \frac{\overset{10}{20} \cdot \overset{6}{19} \cdot \overset{18}{18}}{3 \cdot 2 \cdot 1} = 19 \cdot 60 = 20 \cdot 60 - 60 = 1200 - 60 = \boxed{1140}$$

Or...

$$\frac{\binom{20}{3}}{3!} \leftarrow \begin{array}{l} \# \text{ of ways to choose } 3 \\ \text{where order matters} \end{array}$$

$3! \leftarrow \# \text{ of ways to order the } 3$

$$= \frac{20!}{(20-3)! 3!} \leftarrow \begin{array}{l} \# \text{ of ways to choose } 3 \\ \text{where order doesn't} \\ \text{matter} \end{array}$$

$$\underbrace{\binom{n}{r}}_{\text{"n choose r"}} = \frac{n!}{\underbrace{(n-r)! r!}} \leftarrow \begin{array}{l} n, r \text{ integers} \\ 0 \leq r \leq n \end{array}$$

of ways to choose r objects from n objects, where order doesn't matter

If n, r are integers and $r < 0$ or $r > n$, } convention
then $\binom{n}{r} = 0$.

$$\bullet \binom{n}{0} = \frac{n!}{(n-0)!0!} = \frac{n!}{n! \cdot 1} = 1$$

$$\bullet \binom{n}{1} = \frac{n!}{(n-1)!1!} = \frac{n \cdot \cancel{(n-1)!}}{\cancel{(n-1)!}} = n$$

$$\bullet \binom{n}{n-r} = \frac{n!}{\underset{r}{(n-(n-r))!} (n-r)!} = \frac{n!}{r!(n-r)!} = \binom{n}{r}$$

e.g. $\binom{9}{2} = \binom{9}{7}$, also $\binom{n}{n} = 1$, $\binom{n}{n-1} = n$

Binomial Theorem

$$(x+y)^n = \sum_{r=0}^n \binom{n}{r} x^r y^{n-r}$$

$\binom{n}{r}$ also called a "binomial coefficient"

Expl 1.4d

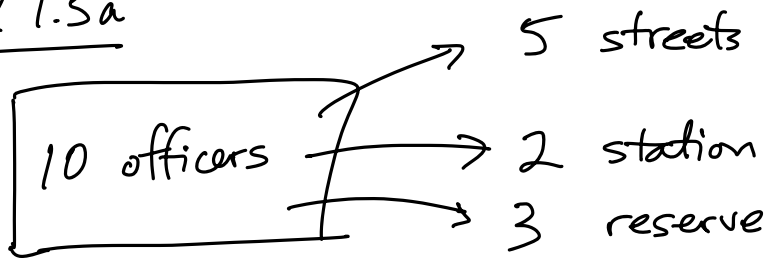
$$(x+y)^3 = \sum_{r=0}^3 \binom{3}{r} x^r y^{3-r}$$

$$= \binom{3}{0} x^0 y^3 + \binom{3}{1} x^1 y^2 + \binom{3}{2} x^2 y^1 + \binom{3}{3} x^3 y^0$$

$$= y^3 + 3xy^2 + 3x^2y + x^3$$

1.5 Multinomial coefficients

Expt 1.5a



How many assignments?

$$\binom{10}{5} \binom{5}{2} \binom{3}{3} = \frac{10!}{5!5!} \cdot \frac{5!}{3!2!} \cdot \frac{3!}{1!3!}$$

↑ choose streets ↑ from the remaining, choose station ↑ rest go on reserve

$$= \frac{10!}{5!2!3!}$$

OR

$$\binom{10}{2} \binom{8}{3} \binom{5}{5} = \frac{10!}{8!2!} \cdot \frac{8!}{5!3!} \cdot \frac{5!}{1!5!} = \frac{10!}{2!3!5!}$$

same

Notation: $\binom{10}{5, 3, 2} = \frac{10!}{5!3!2!}$

of ways of breaking 10 objects into 3 distinct groups (group of 5, 3, and 2)

more on this later

$$\frac{10!}{5!3!2!} = \frac{10 \cdot 9 \cdot \overset{4}{\cancel{8}} \cdot 7 \cdot \cancel{6}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{2}} = 10 \cdot 63 \cdot 4$$
$$= 10 \cdot 252 = \boxed{2520}$$

In general,

multinomial coefficient

$$\binom{n}{n_1, n_2, \dots, n_r} = \frac{n!}{n_1! n_2! \dots n_r!}$$

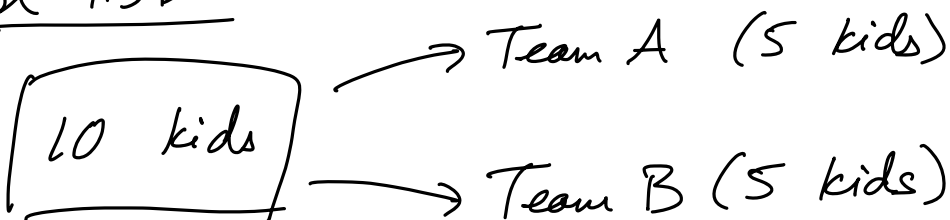
must add up to n

Also,

$$\binom{n}{n_1, n_2} = \binom{n}{n_1} = \binom{n}{n_2}$$

because $n_1 + n_2 = n \Rightarrow n_1 = n - n_2$ and $n_2 = n - n_1$

Expl 1.5b



How many assignments

Teams A and B are meant to be distinct.

e.g. A: red jerseys
B: blue jerseys

e.g. oldest in red, youngest in blue \neq oldest in blue, youngest in red

Because distinct, can use mult. coeff.

