# Randomness in Science

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Let me begin by saying that I am by no means a historian. But one thing I have gleaned from my meager exposure to the subject is that there was once a time in history when science was regarded as just another branch of philosophy. It was called "natural" philosophy, perhaps to distinguish it from "supernatural" philosophies such as theology or metaphysics.

Today, of course, we regard science as a discipline which is quite distinct in character from philosophy. And it is right that we do so. For example, science produces technology, but philosophy does not. So they truly are separate disciplines. And what distinguishes science from philosophy is obvious. There is only one thing: the scientific method. Probably everyone is familiar with the scientific method. I know that when I was in elementary school, we would open our science book, and there it would be, in bold letters, prominently displayed in a boxed area. It was some sort of six-step procedure. There was an acronym for remembering it which contained a "Q" and, I think, three or four "R"'s. It was like the Ten Commandments, laid out in stone, as though it had been there since the dawn of time. But whatever the case may be, the scientific method is just an idea, and we all know what that idea is. It's the idea that we should formulate testable hypotheses and then perform experiments to see whether or not those hypotheses are correct.

Now, a lot goes into forming a testable hypothesis. But the first thing you need – the absolutely necessary ingredient to formulating a testable hypothesis – is that it be phrased in language which is completely unambiguous. You can't, for example, as a chemist, have a hypothesis in which you say, "When I mix A and B, there will be a reaction, and that reaction will be 'very strong'." That just won't work. You would perform the experiment, see the reaction, and say, "Yeah, that was very strong." But the guy next to you says, "Well, it was strong, I'll give you that. But 'very' strong? I don't think so." You have an argument and at the end of the day, you've proved nothing. So before a hypothesis has a chance of being testable, it must be formulated in language that cannot be misinterpreted and does not depend on anyone's opinions. We have a language for that and it's called mathematics. So, in the end, it's really the use of mathematics that separates science from philosophy.

That said, let's return to the beginning of this discussion, which was that science used to be just another branch of philosophy. We might now wonder why. Surely they had mathematics. Mathematics has been around a lot longer than modern science. Why didn't they use their mathematics and create science? Well, let's think about this for a moment. What kind of mathematics did they have? They knew how to count and add and multiply. They knew about prime numbers and rationals and irrationals. So they had a kind of number theory. Geometry had been around since before Euclid. Pythagoras dealt with triangles, so they had trigonometry. But all of these are subjects that deal with *static* objects. When you study trigonometry, you study a triangle. What does it do? Nothing. It just sits there. You look at it and deduce properties of it, but nothing is moving. Nothing is changing. It wasn't until the advent of calculus that we had a branch of mathematics capable of describing the *dynamic* world around us. And everything we care about, everything we want to say and know about the world around us, relates to how it changes and what rules it follows as it makes those changes. Think about our rockets and missiles and satellites, our bridges and buildings and airplanes, everything electronic or magnetic in nature. All of these things we understand and can control because of calculus. The birth of calculus ushered in this era of modern science and, since then, calculus has shaped the world in which we live.

We might ask, then, what *part* of calculus is it that's performing this miracle for us? Is it the idea of the derivative? Or the concept of the integral? Or is it just the whole package altogether? Well, I would like to claim that it is the differential equation. The differential equation is the thing that allows us to translate our hypotheses about the world into mathematics. For example, suppose you want to assert that the acceleration of a falling body is constant, independent of the body's mass or its height or how long it's been falling. Well, what you're really saying is $y'' = g$. Calculus then gives you a way to solve this equation. What this means is that, if your assertion is true, then the falling body ought to behave according to the solution you've just constructed. You can then observe the falling body and see if you're right.

As another example, imagine a bunch of people at a party, getting wild and crazy. Someone attaches a spring to the ceiling and puts a weight at the end of the spring. They pull down on the weight and let it go, watching it bob up and down, giggling and drinking. Just then, someone comes in and says, "You know what? I'll bet that, at any given moment, the net force on that weight is proportional to its displacement from equilibrium."

Everyone looks at him and says, "Huh? What did you say?"

"Well," he replies, "what I really mean is $y'' = -ky$."

"Ahhh! Okay, now that makes sense!" Again, calculus gives us a way to solve this equation and we can observe the bobbing weight and check to see if its motion agrees with the solution. It does, of course. Everyone's happy and the newcomer to the party is regarded as a scientific genius.

Now, I want to give one last example and this will segue into what I really want to talk about. Imagine someone comes to you and says, "Hey, check this out. I've just discovered radioactivity. See this blob of glowing goo? It's radioactive. And I've invented this box that measures the level of radioactivity."

"Great," you tell him. "That's fantastic."

"But wait," he says. "I've observed that the level of radioactivity is decaying over time. And I would like to postulate that, at any given moment, the rate at which it is decaying is proportional to the amount of radioactivity present at that moment." Well, what he's really saying, of course, is that $y' = ry$ for some (negative, in this case) constant $r$. You solve this equation and see that he's right. You congratulate him for being such an insightful genius and that's that.

But there's a subtle problem with this example. You see, this guy doesn't *really* understand the nature of radioactivity. The blob of goo is radioactive because it contains many tiny radioactive particles. The radioactivity is decreasing because those particles are leaving. But exactly when particles will leave, no one can predict – it's random. And, when they do, exactly how many particles will leave, no one can predict – it's random. So the *actual* level of radioactivity is a random process. It is *not* described by the solution to $y' = ry$. What this solution *does* describe is the *average* level of radioactivity, averaged over many different blobs of goo. The actual radioactivity level has random perturbations about this average.

In this example, however, the particles are so tiny, and there are so many of them, and they move so quickly, that these random perturbations cannot be seen. The average behavior is all that can be observed. So from the point of view of the experimenters, $y' = ry$ is a complete description of the phenomenon. We see then, that in this example, the subtle difficulty is, on a practical level, completely irrelevant, and calculus survives as the all-powerful tool of science.

But now enter the biologist. The biologist says, "I am studying this population of micro-organisms and the population is increasing because they are reproducing. I would like to postulate that the population obeys the equation $y' = ry$." All right, fine. So the equation is solved and the population is observed, and suddenly there's a problem. Clearly, the population is not behaving like the solution. It *almost* does, but there are visible random fluctuations. The physicists tell the biologist, "Well, I'm sorry. You're wrong." And the biologist replies, "But I've accurately described the average behavior. This gives me a pretty accurate qualitative description of what's going on." And the physicists say, "All right, fine. That's probably the best you can hope for anyway. And it's only the qualitative understanding that's important to *you*. After all, everybody knows biology's not an exact science. At least not as exact as physics." And that's the end of that. The biologist goes on her way and is left with the stigma of being a "soft" scientist.

Well, things get even worse now. Enter the economist. He says, "I've been looking at the price of gold. It seems to go up by about the same percentage every year and I would like to postulate that the price of gold obeys the equation $y' = ry$." The price of gold, of course, looks nothing like the solution to this equation. If you've ever seen graphs of the price of a stock or an exchange rate on the news or on the internet, you know that these graphs are wild, zig-zag lines. If you squint your eyes and look at the general shape of the graph, you *might* be able to see the solution that the economist is looking for. So perhaps the economist has accurately described the average behavior of the price of gold. But this time, the random fluctuations are not only visible, they're significant, and constitute a major part of the behavior of the process. More than that, though, the economist *cares* about these random fluctuations. He's not just interested in a qualitative description of what's going on. He wants to get his hands on these variations about the average. So the differential equation is even less effective here, and economics is also labelled a "soft" science.

Now I want to tell you a story about two men in the 1970's. Their names were Fischer Black and Myron Scholes. They were looking at this equation, $y' = ry$, as a model for the price of a stock. They knew that this could only represent the average behavior of the stock, but they wanted to study aspects of it beyond the average. To do this, they utilized what was then, and still is, a relatively modern tool in mathematics: Ito's stochastic calculus. Ito is the name of the mathematician who did the major work to develop the calculus, and "stochastic" simply means "random". Black and Scholes knew that the simple equation $y' = ry$ only described the average behavior because $r$ was supposed to represent the average rate of change. To get an equation describing the actual behavior, they wanted to replace $r$ with the *actual* rate of change of the stock. There are two problems with this. First, the actual rate of change depends on time. Sometimes it's large, sometimes it's small. Sometimes it's even negative. So they needed to replace $r$ with a function of time. Of course, that was no problem. Physicists had been doing that for centuries. But the other problem is that the actual rate of change of the stock is random. So they needed to replace $r$ with a *random* function of time. More precisely, they changed their equation to $y' = [r + W(t)]y$, where $W(t)$ is a random function of time, sometimes called "white noise". In order to solve this equation, they used Ito's stochastic calculus. But since the equation involves a random function, the solution itself is a random function. So now they were left with the daunting task of trying to compare the *actual* behavior of the stock with a solution which was only a random function. How was that to be done?

To better understand this difficulty, let's take a step back for a moment and ask ourselves, what kinds of hypotheses can they form based on their solution? With ordinary calculus, they could only make assertions regarding the average behavior of the stock. With stochastic calculus, they could make almost any assertion they could imagine. But the catch is that every assertion comes with a probability. For example, they could say that the stock will go up tomorrow ... with a 15% chance. Or they could say that there's a 60% chance the stock will reach the level 100 within the next 6 months. Are these assertions testable? Suppose they say there's a 15% chance the stock will go up tomorrow. Well, you watch the stock. Either it goes up or down. Were they right? Who's to say? And you can't go back in time and watch that day again. So, on the surface, it seems these assertions are not testable and, scientifically speaking, nothing has been accomplished.

Well, now I'd like to tell you about my grandmother. My grandmother is ninety-five years old. She goes to bed at about seven o'clock. Every night, she watches the five o'clock news while she eats her dinner. She must have been watching these same news channels for decades now. And she will tell you (I can't remember the exact channels, so I'll make them up) that the weatherman on channel 4 is the best and the guy on channel 5 isn't worth a hill of beans. How does she know? The weatherman comes on and says there's a 60% chance of rain tomorrow. Either it rains or it doesn't. Was he right? Who's to say? And you can't go back in time and watch that day again. But, of course, he comes on TV day after day, making his claims. And, somehow, my grandmother has intuitively analyzed all this data and come up with an opinion as to which weatherman is better.

In the same way, the Black-Scholes model can make predictions day after day. Faced with two competing models, it's likely that, over time, even my grandmother could tell you which model was the better one. So our human brains are capable of evaluating probabilistic assertions, given enough of them, and it would be inaccurate to say that these assertions (or, rather, collections of such assertions) are not testable. Exactly what is the right algorithm for testing them? Well, my grandmother appears to know it, at least subconsciously. Perhaps it's just a matter of time before someone formalizes it, writes it down, and it appears as a six-step procedure in elementary school textbooks called the "probabilistic scientific method".

Are we living in a new and expanding scientific era, in which the mathematics of probability is as important as traditional, deterministic mathematics? Can this new era harden the "soft" sciences of yesterday? To help answer this, it should be mentioned that the Black-Scholes method is now a major part of economics, not just in theory, but in practice. It has literally sparked a revolution and has mathematized economics (or at least a part of it) in a way that it had not been mathematized before. There is an extensive and formal mathematical theory of markets and market completion and arbitrage all built upon Ito's stochastic calculus. Similarly, stochastic calculus, and probability in general, has mathematized biology in a revolutionary way. Not only is probability used to study the physical growth and movements of actual organisms, but also to study the virtual movement of DNA through the generations. Genes move in a random way through a family tree and these random "paths" are studied with probabilistic techniques. It may, right now, just be science fiction, but perhaps one day probability can mathematize subjects such as sociology and political science. Who knows, maybe someone reading this right now will be the first person to study politics in the Middle East using stochastic calculus. Will they make peace or predict doom? We'll just have to wait and see.