

Use the  $\varepsilon$ - $\delta$  definition of continuity and uniform continuity in each of the following problems.

1. Define  $f : \mathbb{R} \rightarrow \mathbb{R}$  by  $f(x) = x^2$ . Prove that  $f$  is continuous at  $x = 2$  and  $x = 50$ .
2. Define  $f : [0, \infty) \rightarrow \mathbb{R}$  by  $f(x) = \sqrt{x}$ . Prove that  $f$  is continuous at  $x = 4$  and  $x = 100$ . (See Exercise 3.5.2 for a hint.)
3. Define  $f : \mathbb{R} \rightarrow \mathbb{R}$  by  $f(x) = x^3$ . Prove that  $f$  is continuous.
4. Define  $f : \mathbb{R} \rightarrow \mathbb{R}$  by

$$f(x) = \begin{cases} x + 1 & \text{if } x \leq 3/4, \\ 2 & \text{if } x > 3/4. \end{cases}$$

Prove that  $f$  is not continuous at  $x = 3/4$ .

5. Define  $h : \mathbb{R} \rightarrow \mathbb{R}$  by  $h(x) = 1/(1 + x^2)$ . Prove that  $h$  is uniformly continuous.