HW 5, due Wednesday, February 19, 2014

1. Exercise 2.1.1.

2. Exercise 2.1.2.

3. (i) Let P be a probability measure. Prove that $A = \{\omega : \mu(\{\omega\}) > 0\}$ is countable.

   (ii) Let $(X, \mathcal{A}, \mu)$ be a measure space and $f : X \to [0, \infty]$ a measurable function. Prove that if $S = \{x : f(x) > 0\}$ is countable, then

$$\int_X f \, d\mu = \sum_{x \in X} f(x)\mu(\{x\}).$$

4. Exercise 2.1.8.

5. Exercise 2.1.13.

6. Exercise 2.1.14.